Proceedings of ICIHCS 2025 Symposium: Voices of Action: Narratives of Faith, Ethics, and Social Practice
DOI: 10.54254/2753-7064/2025.ND27892

Sharp Boundaries in a Fuzzy World: Why Epistemicism Survives the Sorites

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Abstract. The sorites paradox reveals the tension between the vagueness of natural language and the principles of classical logic. It has sparked significant contemporary debates in logical philosophy. Traditional binary logic struggles to face the challenges posed by ambiguous predicates. Researchers have proposed various solutions, such as degree theory, supervaluationism, and many-valued logic, but all face interpretive limitations. This paper systematically examines the logical structure of the sorites paradox and focuses on Williamson's epistemicist theory. Epistemicism maintains that ambiguous predicates possess objective and precise boundaries, though these lie beyond human cognitive reach and remain unknowable. By introducing the notion of "unknowability," this theory preserves the validity of classical logic while offering a unique framework for addressing vagueness. This paper analyzes the mathematical logic foundations of epistemicism and its applied value in natural language analysis, legal reasoning, and ethical judgment, highlighting its advantages in theoretical simplicity, interpretive consistency, and intuitive rationality. Although epistemicism still faces challenges such as boundary arbitrariness and higher-order fuzziness, it has significant importance in resolving the sorites paradox and promoting interdiscipcolinary exchange. Finally, this paper proposes that future studies on epistemicism should focus on these three aspects: formal expansion, comparative studies, and interdisciplinary integration.

Keywords: Sorites paradox, vagueness, epistemicism, bivalence

1. Introduction

The Sorites paradox reveals itself as an untoward dilemma in logic and philosophy. First proposed by the ancient Greek philosopher Epicharmus, this paradox has confounded logicians and philosophers for centuries. It reveals the difficulties fuzzy concepts impose on logical reasoning. The paradox can be seen as a process of reduction that exposes the limitations of natural language. For instance, when removing sand grains one by one, it becomes difficult to determine the precise point at which the pile of sand ceases to be called "a pile of sand." Such ambiguity extends beyond predicates of physical objects; it is present in areas like color classification and the application of laws [1].

The challenge of vagueness is both universal and fundamental, exhibiting its most typical characteristics in the interaction between mathematical logic and natural language. Classical logic,

DOI: 10.54254/2753-7064/2025.ND27892

rooted in the law of the excluded middle and the law of non-contradiction, demands propositions be either true or false, allowing no third possibility. However, when this binary logic is applied to natural language expressions containing vague predicates, it encounters difficulties. For instance, is a person 175 centimeters tall considered “tall”? Different standards yield different answers. Similarly, a person with 100,000 hairs is termed “not bald.” If they lose one hair, can they still maintain the label “not bald”? This question is difficult to affirm or deny.

Faced with the Sorites paradox, thinkers proposed diverse solutions: degree theory handles fuzziness through continuous truth-value levels; supervaluationism examines various possible precise definitions while preserving classical logic’s validity; while many-valued logic expands truth-value systems to accommodate fuzziness [2]. Among these theoretical approaches, Timothy Williamson’s epistemicism is outstanding for its unique perspective and argumentation, which help him gain wide attention in contemporary analytic philosophy.

Epistemicism’s basic claim is that fuzzy predicates do have precise boundaries, but they lie beyond the scope of human cognition and thus are unknowable. This view preserves classical logic’s principle of bivalence while offering a rational explanation for fuzziness [3]. Williamson demonstrates through a systematic series of arguments that epistemicism surpasses other theories in both simplicity and explanatory power.

This paper aims to explore epistemicism’s theoretical advantages in addressing the sorites paradox and reveal its significant role in the development of contemporary philosophy of logic. The research systematically examines the logical structure of the sorites paradox, the mathematical logic foundations of epistemicism, and its applications, demonstrating that epistemicism provides satisfactory solutions to issues of fuzziness at both theoretical and practical levels [4]. Methodologically, this paper employs classical analytical philosophy approaches, integrating formal logic with conceptual analysis to systematically review relevant philosophical arguments, thereby offering new insights into core philosophical problems.

2. The logical structure of the sorites paradox: a formal analysis

The fundamental form of the sorites reasoning can be constructed as a deductive reasoning process, whose paradoxical conclusion arises from the logical efficacy of induction. Taking the classic sorites paradox as an example, its argumentative process can be formally transformed into logical steps. First, establish the initial hypothesis: if a set contains a large number of sand grains, it can be called a sand pile. Second, suppose that removing a single grain of sand from the pile does not alter its essential property as a sand pile. Finally, following the general principle of induction, we deduce the conclusion that “a single grain of sand also constitutes a sand pile.” This conclusion clearly contradicts intuitive common sense [5].

Scholars have attempted to precisely express fuzzy predicates within a first-order logic framework but encountered fundamental limitations in expressive power. The idea of the sand pile argument can be summarized as the relationship between the predicate and the number of grains of sand. Let predicate H(x) denote “x constitutes a sand pile,” and n denote the number of grains. This can be formalized as:

$$H(100000) \wedge \forall n(H(n) \rightarrow H(n-1)) \rightarrow H(1) \tag{1}$$

This logical formula is drawn by standard rules like universal instantiation and conditional inference, which makes it fully valid within the classical first-order logic. The problem lies in the

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premise, H(1000000) and $\forall n(H(n) \rightarrow H(n-1))$, which appear intuitively reasonable. However, the conclusion H(1) obviously contradicts common sense.

The paradox arises from the conflict between the bivalent nature of truth functions and ambiguous borderline cases. Within traditional logic, atomic propositions must possess unambiguous truth values, and predicates like H(n) cannot tolerate intermediate states or uncertainty. However, when faced with the proposition “Do 999,999 grains of sand constitute a pile?”, an absolute judgment proves impossible. Intuitively, this question is difficult to verify. It suggests potential flaws in classical logic’s binary nature. Nevertheless, this ambiguity may merely be superficial; precise boundaries could still exist at a deeper level [6].

From a set-theoretic perspective, the distinction between fuzzy sets and classical sets primarily lies in how boundaries are defined. Zadeh’s fuzzy set theory permits degrees of belonging that take values within the interval [0,1]. This membership function effectively addresses the issue of vague boundaries. For instance, an object containing one million grains of sand has a degree of 1.0, one containing only one grain has a degree of 0.0, and objects between these extremes possess degrees between 0 and 1. While this approach technically handles vagueness, it raises the subsequent question how this membership function should be determined and whether it possesses an objective basis.

In contrast, classical set theory employs strict binary definitions: an object either fully belongs to a set or does not belong at all, with no intermediate possibilities. If the sand pile concept is regarded as a classical set, there must exist a critical point where removing a single grain causes the object to transition from “belonging” to “not belonging.” However, this point often appears arbitrary and lacks rationality, making it difficult to explain why precisely this grain of sand—and not an adjacent one—plays the decisive role [7].

From a logical perspective, the appeal of the sorites paradox lies in its demonstration of powerful proof through mathematical induction. Mathematical induction, as a foundational method, relies on the property of natural numbers. The inductive step in the paradox, $\forall n(H(n) \rightarrow H(n-1))$, asserts: if n grains of sand form a pile, then n-1 grains should also form a pile. This assertion has strong intuitive appeal, as removing a single grain of sand seems insufficient to alter the overall property, making it appear particularly reasonable. However, when combined with the initial hypothesis H(1000000), the inductive step leads to an apparent absurd conclusion, thereby revealing the tension between logical reasoning and intuitive cognition [8].

3. The mathematical-logical foundations of epistemicism

In addressing the sorites paradox, Williamson’s epistemicism adopts a theoretical approach that is both bold and robust. Its boldness is the insistence that fuzzy predicates necessarily possess precise boundaries, though these boundaries transcend human cognitive capacity and thus remain unknowable; its robustness lies in preserving fundamental principles of classical logic, including bivalence and the law of the excluded middle. The key claim of epistemicism can be formulated as follows: for any fuzzy predicate F and sequence of objects x_1, x_2, …, x_n, there exists a definite boundary point k such that $F(x_i)$ is true if and only if $i \leq k$. However, the precise location of this boundary point k cannot be known by humans.

Within the framework of modal logic, the knowability operator K can be introduced to formally express the insight of epistemicism. If Kφ denotes “the proposition φ is knowable,” then epistemicism holds that: although the boundary of a fuzzy predicate does indeed exist, its truth value cannot be grasped by the cognitive subject. Taking the sandpile predicate H as an example,

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epistemicism acknowledges the existence of a natural number k such that H(n) is true when $n \geq k$. Simultaneously, it asserts ¬K(H(k)), meaning the truth value of H(k) cannot be known with certainty. This formalized formulation balances metaphysical precision with epistemological limitations, thus offering a novel resolution to the Sorites paradox [9].

The theory of knowability plays a crucial role within this framework. Williamson's argument demonstrates that if the boundary of a fuzzy predicate were knowable, it would generate new paradoxes. For instance, suppose there exists a smallest natural number k such that k grains of sand do not make a sand heap, while k+1 grains do, and this fact is knowable. It can be formalized as:

$$K(\neg H(k) \land H(k+1)) \tag{2}$$

This raises the question as to why precisely k grains of sand serve as the boundary, rather than k-1 or k+1? Explaining this requires reliance on minute physical distinctions that exceed human cognitive resolution capabilities, making their reliable identification impractical.

The phenomenon of higher-order fuzziness further supports epistemicism. This phenomenon implies not just that the original vague predicate has an uncertain boundary, but that the metalinguistic description of this kind of boundary is equally vague. In other words, fuzziness can stretch to higher levels, forming second, third, or even higher-order fuzziness. Within the epistemicist framework, each level of fuzziness corresponds to a degree of unknowability. Not only are the precise boundaries of fuzzy predicates unknowable, but the extent of the boundary regions themselves is equally unknowable. It thus constitutes a recursive structure of uncertainty.

From a semantic perspective, epistemicism seeks to establish a balance between precision and unknowability. Traditional semantics claims that the meaning of linguistic expressions should be entirely clear under ideal conditions. Epistemicism, however, offers an alternative semantic perspective: linguistic expressions do have precise meanings and definite truth-conditions, but these conditions exceed the grasp of human cognition. This view can be formalized through mathematical modeling, where the semantic function maps expressions to truth-states, but finite cognitive agents cannot fully grasp these truth-states.

Williamson further argues that epistemicism not only solves the Sorites paradox but also offers a unified explanation for other philosophical problems. Concerning observables, epistemicism reveals their fundamental dependence on observation, making precise application conditions inherently elusive. For example, regarding the predicate “red”, while colors are commonly perceived as directly discernible through vision, the transition zone between red and orange defies precise classification even under the most meticulous observation. Epistemicism explains this by claiming that the boundary of red does exist, but its exact location lies beyond the resolving power of the human visual system [1].

4. The application of epistemicism in philosophical reasoning and natural language analysis

Epistemicism not only demonstrates explanatory power in logic and philosophy but also exhibits broad practical value in related fields such as natural language analysis. A central challenge long faced by linguists and philosophers is how ambiguous predicates can support effective communication without clear conditions of application. For this, epistemicism offers an explanatory framework. Its fundamental view is that ambiguous predicates do contain precise semantic content, but speakers in actual communication do not have to master boundaries. Instead, they attain understanding through consistent judgments across most unambiguous contexts.

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This theory elucidates speakers' behavior when using ambiguous predicates. For instance, while definitions of "tall person" may vary intuitively, consensus exists on extremes, since someone 2.2 meters tall is unquestionably tall, whereas someone 1.2 meters tall clearly is not. Epistemicism explains that while the precise boundary of "tall" exists yet remains unknowable, people form approximate cognitive judgments based on experience and intuition. These judgments are relatively reliable when they are far from the boundary, but become more prone to disagreement and uncertainty as one approaches the boundary.

Legal reasoning and moral judgment offer particularly practical applications for epistemicism. Many key concepts in legal systems, such as "reasonable doubt," inherently have ambiguity. Applying these concepts in specific cases often demands complex balancing acts. Traditional jurisprudence typically views this as an inherent vagueness of legal language, tackling it through graded standards or multi-factor balancing mechanisms. Epistemicism proposes an alternative interpretation according to which legal concepts possess precise conditions for application, but these conditions are too complex for human cognitive resolution. Thus, judicial practice only needs approximate judgments and heuristic methods [10].

In ethics, the application of epistemicism is illuminating as well. Many moral concepts inherently carry ambiguity. They are often invoked to support moral relativism or subjectivism. Epistemicism, however, offers a new defense for moral objectivism. It believes moral concepts do possess objective and precise conditions for application, determined by moral facts independent of human cognition. Due to the limitations of human cognition, it is hard to fully grasp these conditions, and this fact leads to divergences in actual judgments. This perspective retains the objectivity of moral judgment while explaining the uncertainty in moral disputes.

A crucial component of epistemicism is its critique of alternative theoretical frameworks. Regarding degree theories, Williamson highlights the precision paradox. If a proposition has a truth value of 0.7, one must then ask "Is 0.7 itself precise?"—triggering second-order paradoxes or infinite recursive truth-value questions. Despite introducing continuous truth values, degree theories fail to resolve the problem of vagueness. In contrast, epistemicism can avoid recursive dilemmas by adhering to the principle of bivalence.

Concerning supervaluation theory, epistemicism's critique centers on its "artificiality" and "unnaturalness." Supervaluation theory maintains the validity of classical logic by examining all possible precise definitions, but this approach fails to reveal the true semantics of fuzzy predicates. It merely preserves the logical system through technical means. Particularly when handling higher-order fuzziness, the over-determination theory necessitates the continuous introduction of new precision operations, which results in an increasingly complex and unintuitive theoretical framework. In contrast, epistemicism offers a more precise explanatory framework.

At the formal level, epistemicism models fuzziness by means of functions and algebraic structures. Specifically, fuzzy predicates can be taken as characteristic functions χF, whose domain is the set of relevant objects. Although χF is mathematically well-defined, its precise definition remains unclear to those with finite cognition. This mathematical expression provides epistemicism with precise theoretical tools while also laying the foundation for computational modeling and interdisciplinary research.

5. Conclusion

Through systematic examination and detailed analysis of epistemicism, this paper concludes that epistemicism demonstrates remarkable explanatory power and theoretical advantages in addressing the sorites paradox. This strength stems from its unique logical position and methodological

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orientation. On one hand, it upholds pivotal principles of classical logic such as bivalence and the law of the excluded middle; on the other hand, it introduces the concept of unknowability to provide a reasonable explanatory framework for phenomena of ambiguity. Thus, epistemicism effectively addresses the challenges posed by natural language without departing from the classical logical framework.

The strengths of epistemicism lies primarily in three aspects: theoretical simplicity, interpretive consistency, and intuitive rationality. However, epistemicism is not without challenges. The primary issue is the arbitrariness of boundaries. If fuzzy predicates have precise demarcation points, why does one particular point become the boundary while its neighboring points lack an equivalent status? For example, if the determination of "baldness" relies on a specific number k, it becomes difficult to explain why precisely k hairs play the decisive role. Epistemicists respond: the apparent arbitrariness of boundaries does not imply their non-existence. The unidentifiability of boundaries does not deny their objectivity. Another significant challenge is about the treatment of higher-order fuzziness. Epistemicism explains higher-order fuzziness through "recursive unknowability," but this strategy merely shifts the problem to a higher level rather than resolving it. If the metalinguistic framework itself has precise boundaries, it still faces arbitrariness issues and may even trigger infinite recursion. Therefore, epistemicism must clarify reasonable termination conditions for recursive processes to avoid theoretical overload.

Nevertheless, epistemicism remains valuable in contemporary logic and philosophy. It not only provides a robust solution to the Sorites paradox but also establishes a theoretical platform for interdisciplinary research. Its emphasis on unknowability offers opportunities for integrating formal semantics with cognitive science. In the future, the developmental prospects of epistemicism manifest primarily in three directions. First, it should integrate with mathematical tools such as probability theory and information theory to achieve more precise quantification of uncertainty. Second, at the interdisciplinary level, research combining artificial intelligence and cognitive science can be strengthened to advance the application of epistemicism in practical and technological fields. Third, comparative studies with other theoretical approaches to vague predicates should be deepened to enhance the explanatory power of epistemicism within broader contexts.

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