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The Puzzle of Informative Identity: A Descriptivist Response

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Abstract. The puzzle of informative identity has been a widely debated logical issue for a long time. To be more specific, the problem of how people should define the exact referent of identity statements such as "A is B" remains mysterious. When the same object is referred to by different names, this kind of identity statement seems a priori and non-informative. In addition, defining identity statements as connections between "names" would mean that they are relations between symbols. This nature of identity statements makes them an urgent philosophical problem. This paper argues that Bertrand Russell's descriptivist theory provides a resolution to Gottlob Frege's puzzle of informative identity statements by fundamentally dissolving its core assumptions, rather than merely offering an alternative argument. This paper reaches this conclusion by justifying Russellian descriptivism's superiority over Frege's metaphysical theory, which involves sense and reference. This paper defends descriptivism against Kripke's objections by altering the modal functors such as "necessarily" and "possibly" in a sentence in natural languages to show that the "scope" of a sentence is controllable.

Keywords: descriptivism, sense, reference, informative identity, rigid designator

1. Introduction

In Sense and Reference, Gottlob Frege questions the fundamental nature of "identity". When analyzing identity statements, he raises the question of whether identity is a relation between objects themselves or between names of objects [1]. In an identity statement, both names are referring to the same object, which means that the equation can be written in the form of a=a, while if we define an identity statement as the relation between names, then its equation becomes a=b, which has entirely different cognitive value. In Critique of Pure Reason, Kant argues that statements such as a=a are known as analytic judgments, which means that the predicate a=a is already contained within the concept of the subject a, which does not provide new information but merely clarifies the existing concept [2]. However, many identity statements require empirical observations and experiments, which are not self-evidently true [2].

On the other hand, the statement a=b is constructing a relation only between names and signs, and it is arbitrary to infer a connection between this relation and the designated subject. Therefore, "the sentence a=b would no longer refer to the subject matter, but only to its mode of designation" [1]. The puzzle arises since there is an urgency for philosophers to find an approach to interpret identity statements in a way that proves their informativeness. Frege himself attempted to work

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through the puzzle and define the nature of identity statements through creating and emphasising the sense-reference distinction on a metaphysical level, which preserves the subject matter by proposing that co-referring names can present that single object in different ways (through different senses), thereby explaining the cognitive significance [1].

The previous research on Frege's puzzle of informative identity has been divided into two camps. The first, following Frege, accepts the puzzle's terms and seeks a solution within a framework where names possess both sense and reference. The second, influenced by direct reference theorists like Saul Kripke, rejects Frege's solution but often remains within the puzzle's fundamental structure by seeking alternative explanations for the cognitive significance of identity statements. A significant need exists for a persuasive analysis of the theory that fundamentally dismantles the puzzle's premises. This paper argues that Bertrand Russell's descriptivist theory, by analyzing ordinary proper names as disguised definite descriptions, does not merely offer an alternative solution to Frege's puzzle. It effectively dissolves it by rejecting its core assumption that true identity statements in the form a=b state the identity between directly referring expressions. This paper will approach this conclusion by expositing Frege and Russell's arguments for the puzzle of informative identity and arguing in favor of Russell's position as a better and clearer alternative. The paper will rehearse the strongest objection by Kripke to Russell's descriptivist theory and defend Russell's claim from multiple perspectives.

2. Frege's metaphysical solution

In the earlier book Begriffsschrift, Frege develops his elementary idea about the nature of identity statements. He observes that signs such as names or expressions can stand either for their content or for themselves. Identity statements like "a=b" might appear superfluous, as one could assume that no special sign for the identity of content is needed because people would be able to know that a and b refer to the same thing if they understand the referents of a and b, respectively [3]. However, Frege illustrates that the same content can be denoted in different ways. Using a geometric example, he shows that a point on a circle can be identified in two different manners: either directly through intuition or indirectly as a geometrically descriptive sentence. Although both methods refer to the same point or content, the names used in the sentences initially suggest different designations. This necessitates a special sign for the identity of content, which connects two distinct names referring to one and the same object. Frege formalizes this with the notation "$\vdash(A \equiv B)$", meaning that signs A and B have the same fundamental content and can be substituted for one another [3].

This early insight is developed further in Sense and Reference. This latter work of Frege introduces the crucial distinction between sense (Sinn), which is the "mode of presentation" or way of determining an object, and reference (Bedeutung), which is the object itself [1]. This theory goes beyond semantic analysis and interprets the identity of contents on a metaphysical level by explaining the dual nature of the objects we refer to in an identity statement. With this theory, "a=b" is informative because the names "a" and "b" express different senses, even though they share the same referent [1]. This lays the foundation for his solution to the puzzle of informative identity.

3. Russell's descriptivist response

However, Russell gives two main objections towards Frege's metaphysical system of sense and reference in On Denoting, which attempts to prove the contradictions in Frege's sense-reference relationship. His first, and most aggressive, objection is that the theory creates an "inextricable tangle" [4], which suggests that Frege's sense-reference argument fails to consider the case where

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we want to refer to a sense of something, and this makes the definition of sense mysterious. Russell suggests that "when we distinguish meaning and denotation, we must be dealing with the meaning" [4], which, in Frege's definition, is considered as "sense". According to John Searle's interpretation, Russell emphasizes Frege's premise that "(2a) whenever the sense of an expression occurs in a proposition, the proposition refers to the referent of that case" [5].

After this process, Russell argues that "if the sense in question were genuinely referred to it would then become a constituent of the proposition, it would occur in the proposition, and by (2a) if the sense occurs in a proposition reference is then made to the referent of that sense, [...], and we have not succeeded in referring to the sense we wanted" [5]. If one tries to refer to the sense of a phrase (e.g., "the sense of 'the author of Waverley'"), "there is no way of identifying this secondary sense as a function of the original sense, [which means that] there is no way of identifying a sense in terms of its corresponding reference" since there are infinite number of denoting phrases attached to this reference [6]. This "lack of a backward road from denotations to meanings" means Frege's framework provides no coherent account of what senses are, making their logical role unclear and mysterious [4].

Different from Frege, Russell's semantic theory of descriptivism provides a more solid and clear alternative to Frege's metaphysical concepts. In On Denoting, Russell claims that descriptivism successfully resolves the puzzle of informative identity by revealing how names symbolize descriptive content that makes identity statements both true and meaningful. When a statement like "Scott is the author of Waverley" is examined, descriptivism shows that the name "Scott" functions as an abbreviation of a series of descriptions attached to it [4], while "the author of Waverley" serves as a definite description tied to a denoting phrase which can be written in the form of a predicate in first-order logic (e.g. "W: x is the author of Waverley"). With first-order logic, there is no necessity to draw an equals sign between "Scott" and "the author of Waverley". Instead, it can be phrased into the logical formula:

$$\exists_x(\mathrm{W}_x \wedge \forall_y(\mathrm{W}_y \rightarrow y = x) \wedge \mathrm{B}_y \tag{1}$$

and substituting the entity Scott into the variable x can effectively make the sentence meaningful [4].

Meanwhile, this formula gives the sentence a truth value that can be examined in a logical way. To be more specific, in first-order logic, predicates carry their own truth values, and functors such as negation function on the truth value of the predicates, which means that a logical expression possesses its truth value as a whole. Being written in the form of first-order logic, the truth value of an identity statement like "Scott is the author of Waverley" is able to be evaluated. Therefore, this paper argues that meaningfulness and truthfulness can appear on a single identity statement if the descriptivist approach is taken. Russell's theory is a better solution because it avoids the problem that Frege is doubted for having by avoiding vague metaphysical concepts that cannot able to be fully explained.

4. Model objection and an alternative theory from kripke

Another logician, Saul Kripke, points out an objection to Russell's argument for wrongly building inter-defining connections between contingent facts. These facts are true in the world people live but might be wrong in other possible worlds, while necessary facts are true in all possible worlds. This objection works specifically for Russell's propositions that attach a descriptive phrase to a name, which implies that the name carries the property (e.g., "Scott is the author of Waverley").

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Kripke defines names used in such name propositions as rigid designators, which means that "in every possible world it designates the same object" [7]. For example, the name "Scott" refers to the same historical figure of Scott in all possible worlds because no one other than Scott could have been Scott. Thus, proper names such as Scott are rigid designators. The example "Scott is the author of Waverley", according to Russell's descriptivist interpretation, can be explained as the name Scott is the abbreviation of the description 'the author of Waverley', and the two terms are interchangeable [7]. But the problem is that this is not a necessary fact. People are able to contend that the identity of a proper name cannot be considered as absolutely dependent on the descriptions attached to it. In other words, Scott would still be Scott if he were not the author of Waverley. Therefore, proper names do not have the same meanings as non-rigid descriptions. If N is a proper name, and D is a non-rigid description, the sentences F(N) and F(D) do not have the same meaning or express the same proposition [8].

To explain in another way, "if the name means [absolutely] the same as that description or cluster of descriptions, it will not be a rigid designator. It will not necessarily designate the same object in all possible worlds, since other objects might have had the given properties in other possible worlds, unless (of course) we happened to use essential properties in our description [7]." Clearly, the denotations like "Scott" are proper names, but descriptivism defines them as being necessarily and tightly attached to some contingent knowledge by arguing that they are interchangeable, which undermines their identities as proper names. This criticism is fundamentally a worry that if names are the abbreviations of descriptions, "Necessarily, 'Scott' is the author of Waverley" would be considered as being equivalent to "Necessarily, 'the author of Waverley' is the author of Waverley", but the former one can be false while the latter one cannot.

Alternatively, Kripke provides his own resolving approach to the puzzle about informative identity statements, which involves rejecting the Russellian descriptivist account of names as abbreviated descriptions and instead grounding reference in what later came to be called the "causal-historical theory". Kripke's alternative argument should be divided into two cases: equivalence between proper names and a descriptive proposition of a proper name.

For the first case, Kripke argues that "terms as 'Hesperus' and 'Phosphorus', when used as names, are rigid designators. They refer in every possible world to the planet Venus [7]." This means that if the claim "Hesperus is Phosphorus" is a fact in this contingent world, then it is true in every possible world. This is because when people use the names, they can say in advance that if Hesperus and Phosphorus are the same, then they can be different in no other possible world since their references are fixed as we use them as proper names [7]. As Kripke argues, although the statement, "Hesperus is Phosphorus," is necessarily true, it is not a priori. It could only be known by empirical investigation. Thus, the informativeness of the discovery does not depend on a hidden descriptive "sense" but on the fact that we arrive at the identity by empirically constructing a connection between two independent chains of reference.

For the second case, identity statements composed by a series of descriptions and a rigid name fix the reference by some contingent marks of the object [7], which are not definite and necessary as claimed by Russell's descriptivist approach. According to Kripke, people pass on names through history and social interaction, not by description. To be more specific, "the reference seems to be determined by the fact that the speaker is included in a community of speakers using the name." The usage or understanding of a specific name is transferred to the person through previous speakers with the same culture [7].

In conclusion, Kripke alternatively argues that identity statements like "Hesperus = Phosphorus" should be understood as necessary a posteriori truths since they possess rigidity, while their

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informativeness arises from the contingent and empirical process of discovery.

5. Dummett: a descriptivist response

Defending descriptivism against Kripke's criticism, Michael Dummett suggests an argument focusing on the structure and "scope" of sentences in natural language, which explains that the distinction between "proper names" and "descriptions" does not exist.

Michael Dummett rejects Saul Kripke's modal argument by stating that the apparent difference in the ways that proper names and definite descriptions function in a sentence is an illusion based on a failure to apply the same mode of logical analysis to both. Kripke argues that the ambiguity in a statement like "the author of Waverley may not have written Waverley" can be explained by Russell's argument of "denying the existence of something" ("the one and only person who wrote Waverley did not write Waverley" is wrong, but "the one and only person who wrote Waverley is such that he might not have written Waverley" is true). However, a similar statement using a proper name, such as "Scott might not have written Waverley," is different from the previous one because names are designators that do not describe, which means that the results we get would be roughly the same no matter which Russellian path we choose to take.

Dummett fundamentally rejects this difference by declaring that in both cases, there is an ambiguity in modal statements containing them. Since it is the very same ambiguity in both cases, it cannot be used to differentiate the two types of expression [9]. More specifically, Dummett contends that the distinctions for names are merely about the "meaning", not semantics. Dummett emphasizes the concept of "scopes" in Frege, Philosophy of Language, which suggests that an identity statement can be interpreted in either a narrow or a wide way. According to Kripke, proper names can be considered as using a wide scope in ordinary language because their semantic function is to refer directly to the specific object itself, which should be considered differently from the case of denoting phrases. However, this does not mean that narrow-scope interpretations cannot be properly explained. For example, an interpretation with a narrow scope of "Scott might not have written Waverley" would be something like "It is possible that the individual we call 'Scott' did not write Waverley". In this second way of interpretation, the name "Scott" would not necessarily be a rigid designator. Instead, it can be used as a denoting phrase, and in this way of interpretation, there is no difference between the name Scott and a descriptive phrase here.

Thus, the ambiguity between the denotation of the object and the denotation of the statement exists for both names and descriptions. Dummett concludes that Kripke's argument on a metaphysical name-description distinction is unnecessary. The Russellian descriptivist theory can be properly applied in order to explain the nature of both name and description in an identity statement. Even further, since the theory that proper names are abbreviated definite descriptions provides a uniform account for the different kinds of ambiguity, it would not risk falling into the parallel ambiguity [9].

Based on this, Dummett concludes that Kripke is making a mistake by defining proper names as rigid designators, which is essentially different from the non-rigid nature of descriptions. Instead, Dummett claims that their difference is no more than the mode of sentence, where a sentence scope excluding the modal operator would have a scope of definite description, while a sentence scope including the functor in its modal context should always be considered as a proper name. Even if this were the case, it would not demonstrate the non-identity of a proper name with a definite description in a strong sense. It simply shows that they behave differently with respect to ad hoc conventions employed for determining scope [9]. In other words, by contending that a term is rigid, people mean that the sentence is always interpreted as having wide scope relative to modal

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operators. By contrast, to define a term as non-rigid is to say its scope can be varied, which means that it can be read as taking a narrow or wide scope, and this can be modified by semantic structures and is not the purely metaphysical difference that is able to support Kripke's objection.

6. Conclusion

This paper argues that Russellian descriptivism offers a solid foundation for resolving the puzzle of informative identity. In response to the confusing nature of identity-denoting statements, the observer of this dilemma, Frege, offers a metaphysical argument which involves the co-existing and co-functioning concepts of sense and reference. However, this metaphysical attempt fails to combine the metaphysical dimension and the semantic dimension, where it is impossible to explain the specific definition of sense or to refer to the sense of something in an identity statement. Alternatively, Russell suggests that people should treat names as the abbreviation of several denoting phrases when they are analyzing identity statements. This method allows people to properly transform a sentence in natural language into first-order logic, which allows people to examine its truth value. Although Kripke provides a strong objection to Russell's descriptivism by arguing that it wrongly connects the contingent descriptions with necessary names, it can be properly resolved by appealing to Dummett's concept of scopes, which argues that one specific kind of sentence structure can be used for descriptivist analysis without suffering Kripke's criticism. Therefore, the equivalence of "A is B" can be informative and significant at the same time by applying Russell's explanation.

References

[1] Frege, G. (1948) Sense and Reference. The Philosophical Review, 57(3), 209-230.
[2] Kant, I. (1998) Critique of Pure Reason. New York: Cambridge University Press.
[3] Frege, G. (1967) Begriffsschrift. In J. Van Heijenoort (ed.) From Frege to Gödel: A Source Book in Mathematical Logic, 1879-1931. Cambridge: Harvard University Press, 1-82.
[4] Russell, B. (1905) On Denoting. Mind, 114(456), 873-887.
[5] Searle, J. (1958) Russell's Objection to Frege's Theory of Sense and Reference. Analysis, 18(6), 137-143.
[6] Blackburn, S. and Code, A. (1978) The Power of Russell's Criticism of Frege: 'On Denoting'. Analysis, 38(2), 65-77.
[7] Kripke S. (1940) Naming and Necessity. Cambridge: Harvard University Press.
[8] Soames, S. (1998) The Modal Argument: Wide Scope and Rigidified Descriptions. Nous, 32(1), 1-22.
[9] Dummett, M. (1973) Frege, Philosophy of Language. New York: Harper & Row.